AIAA JOURNAL
Vol. 30, No. 7, July 1992

Streamlines, Vorticity Lines, and Vortices Around Three-Dimensional Bodies

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The properties of three-dimensional, steady, vortical flows are studied using both theoretical analysis and computed flowfields. The centerline of the vortex is described using minimum streamline curvature, maximum normalized helicity, and the edge of a separation sheet. Analysis indicates that several criteria must be met for these three descriptions of the centerline to agree. In certain regions of the flow, computed flowfields indicate that they do agree, that the velocity and vorticity fields are aligned at the centerline, and that extrema in the velocity magnitude, vorticity magnitude, pressure, and density occur at the centerline.

Introduction

FLOW separation and vortical flows have received considerable attention through the years due to their effects on many physical systems including pipe flows, telegraph wires, aircraft, and turbulence. Phenomenological, analytical, and topological approaches have all been used to describe vortical flows, and the following two definitions have been widely accepted: a vortex is "a finite volume of rotational fluid, bounded by irrotational fluid or solid walls" (Saffman and Baker[1]) and a vortex core is a region with a finite cross-sectional area of relatively concentrated vorticity (Batchelor,[2] Hall,[3] and others). In three-dimensional separated flows, the vorticity varies smoothly throughout the flowfield, and there are no discontinuities that define the extent of the vortices and their cores. Hence, these definitions require interpretation and are not entirely satisfactory.

Recently, the use of topological ideas and bifurcation theory have provided a mathematically rigorous framework for studying separated flows. In the topological approach, two types of separation, global and local (crossflow), have been identified. Examples of two types of global separations (horseshoe and Werle[4]) and one local separation are shown in Fig. 1. For global separation, there always is a critical point on the surface (saddle point of separation)[5,6] associated with the flow separation. Emanating from this saddle of separation is a line of separation, and intersecting the surface at this line is a separation sheet that extends into the flow. This sheet has an edge in the flow that corresponds to a line emanating either from a three-dimensional critical point in the flow (e.g., horseshoe separation) or from a critical point on the surface (e.g., Werle-type separation). This line is uniquely defined, and for steady flows it is a streamline and represents a practical and precise definition for the centerline of the vortex.

For crossflow separation, there are no three-dimensional critical points on the surface or in the flow associated with separation, and there is no unique streamline corresponding to the edge of a separation sheet. However, downstream of the onset of separation, the flowfield has all of the characteristics of separated flows: there is a strong convergence of the limiting streamlines on the surface and there are concentrated regions of vorticity in the flow. For both global and local separation, vortices in the flowfield can be visually identified with organized regions of fluid where streamlines spiral about a central line. This suggests a natural definition of the vortex centerline: a streamline within a region of spiraling streamlines that has minimum curvature.

This definition of the vortex centerline can be compared with that using normalized helicity (Levy et al.[7]). Levy used Moffat's discussion[8] of helicity and relative helicity (normalized helicity) to develop a method for graphically determining the center of the vortex: the center is defined by a minimum in the angle between the velocity and vorticity fields or, equivalently, by a maximum in the normalized helicity (a maximum in the cosine of the angle between the two fields). Using this definition downstream of the onset of a global separation, Levy et al. were able to trace the center of the vortex back to the vicinity of a three-dimensional critical point in a computed flowfield. Although Levy et al. stated that using the normalized helicity is equivalent to finding a streamline of minimum curvature, this will be shown not to be the case, and the minimum curvature and the maximum normalized helicity may not give identical results for the vortex centerline. However, under some circumstances the centerlines defined by these two methods do agree.

In this paper, we investigate the possibilities of defining a vortex centerline for steady flows using either minimum curvature or maximum normalized helicity. The consequences of these two definitions are examined using analysis where possible. Computations are used to extend the analysis and to verify some conclusions. Computed flowfields contain all of the features of experimental flowfields and allow for extensive investigations of flowfield phenomena when analysis alone is insufficient or when experiments do not provide enough detailed information. Recently, several detailed studies of global and local separations, including those of Ying et al.,[9] Levy et al.,[7] and Yates and Chapman,[10] have made use of computed flowfields.

Using analysis and computations, the vortex centerline described by a minimum in the streamline curvature or by a maximum in the normalized helicity is compared with the

Presented as Paper 91-0731 at the AIAA 29th Aerospace Sciences Meeting, Reno, NV, Jan. 7-10, 1991; received May 8, 1991; revision received Oct. 30, 1991; accepted for publication Oct. 31, 1991.

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streamline defined by the edge of a separation sheet. The location of extrema in the pressure, vorticity, and velocity relative to these lines are discussed. Furthermore, the behavior of vorticity lines in crossflow separation is described.

Theoretical Analysis

If a vortex centerline is defined by a minimum in the streamline curvature, by a maximum in the normalized helicity, or by the edge of a separation sheet, the properties of the centerline are of interest. Is it a streamline? Do other extrema fall on this line? If there is a spiral node in a symmetry plane or on the surface associated with the vortex, does this line trace back to the spiral node? These are all questions that should be answered with either analysis or computation. As previously mentioned, a centerline defined by the edge of a separation sheet in steady flow is a streamline and can be traced back to a critical point either on the surface or in the flow. However, there are no uniquely determinable separation sheets associated with crossflow separation, and another definition for the centerline must be developed. In this section, the curvature and normalized helicity definitions are compared with each other, and their relationship to the edge of the separation sheet, when it exists, is discussed.

Critical Points

For any spiral node-saddle in the flowfield or on the surface, there are an infinite number of streamlines that either enter or exit the spiral node-saddle. However, if an infinite number of streamlines approach the spiral node-saddle, only two lines exit it. Conversely, if an infinite number of streamlines exit the spiral node-saddle, only two streamlines approach it.[6] This behavior is a direct result of the continuity

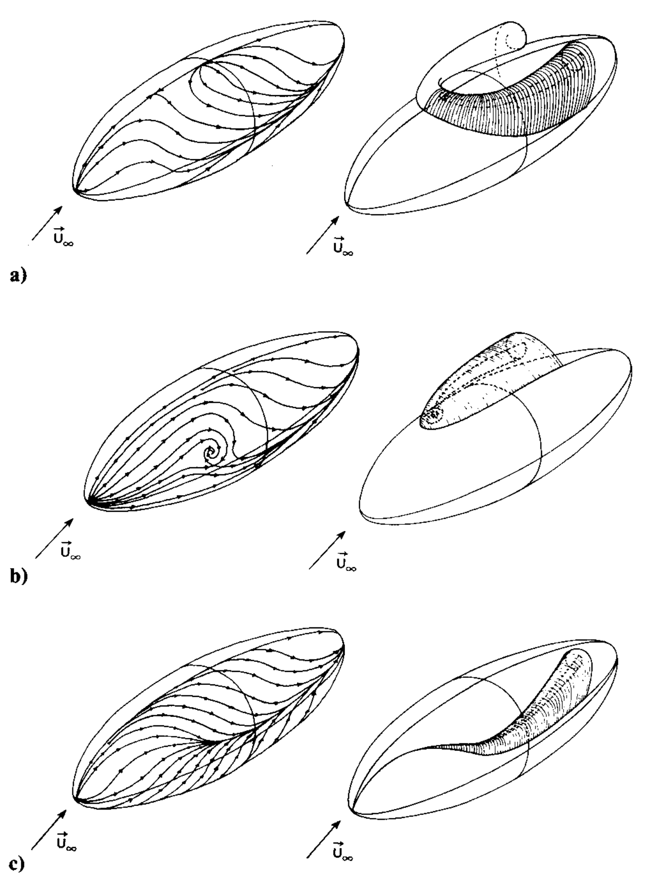

Fig. 1 Limiting streamline pattern and surfaces of separation for three types of three-dimensional separation[10]: a) horseshoe-type separation; b) Werle-type separation (the view of the surface of separation is rotated 90 deg from the view of the limiting streamline pattern); c) crossflow separation.

equation. At any critical point off the surface, the velocity field can be expanded as

$$\boldsymbol{u} = \boldsymbol{A}\boldsymbol{x}$$

or

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

The continuity equation for steady flows, $\nabla \cdot \rho \boldsymbol{u} = 0$, implies that the trace of $\boldsymbol{A}$, $a_{11} + a_{22} + a_{33}$, is equal to zero and that the sum of the real parts of the eigenvalues of $\boldsymbol{A}$ are equal to zero. Hence, if the critical points are spiral node-saddles, two of the eigenvalues, λ_1 and λ_2, are complex and their real parts share the same sign. The third eigenvalue, λ_3, is real and has the opposite sign. An infinite number of streamlines that approach (or exit) the spiral node-saddle can be constructed as long as the eigenvector with eigenvalue λ_3 is not included in the construction. If it is included, the streamline avoids the critical point. However, two streamlines exit (or approach) the critical point, and their direction is given by a positive and negative constant times the third eigenvector. These are the two streamlines that define the edge of the separation sheet if it exists.

It can be shown that in general these two lines do not coincide with a vorticity line and that the normalized helicity is not maximized along these lines. However, under certain conditions, e.g., on a symmetry plane, it easily can be shown that these streamlines and the vorticity lines are aligned and that the normalized helicity is a maximum. First, consider a symmetry plane at $x_3 = 0$. From symmetry arguments, $u_1(x_3) = u_1(-x_3)$, $u_2(x_3) = u_2(-x_3)$, and $u_3(x_3) = -u_3(-x_3)$, and the matrix $\boldsymbol{A}$ can be reduced to

$$\begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}$$

The vorticity at the spiral node-saddle is nonzero and is given by

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ 0 \\ a_{21} - a_{12} \end{pmatrix}$$

The third eigenvalue for $\boldsymbol{A}$ is simply a_{33}, and its eigenvector $\hat{x}_3$ is aligned with the vorticity vector. For vortices cutting through symmetry planes, the nodal behavior is in the symmetry plane, and the vorticity vector is aligned with the streamline that exits the symmetry plane at the spiral node-saddle. At the spiral node-saddle, the limit of the normalized helicity is a function of the direction in which the limit is taken. If the limit is taken in the symmetry plane, the normalized helicity is equal to zero; however, if the limit is taken along the streamline exiting the symmetry plane, the normalized helicity approaches one at the critical point and is maximized.

At a spiral node-saddle on the surface, the streamline and the vorticity line exiting the critical point and extending into the flow are not necessarily parallel. On the surface, the velocity field near a spiral node-saddle can be written as

$$\boldsymbol{u} = x_3 \boldsymbol{A}\boldsymbol{x}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = x_3 \begin{pmatrix} a_{11} & a_{12} & \tfrac{1}{2}a_{13} \\ a_{21} & a_{22} & \tfrac{1}{2}a_{23} \\ 0 & 0 & \tfrac{1}{2}a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

where x_3 is normal to the surface. Satisfying the continuity equation to first order in the x_j identifies the zero terms in the matrix and gives $a_{11} + a_{22} + a_{33} = 0$. The preceding coupled,

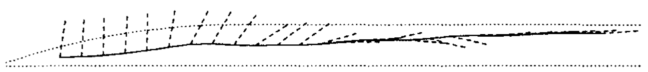

Fig. 2 Change in vorticity direction along a streamline. Dashed line represents vorticity direction.

first-order differential equations can be solved to give the streamline equations:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \alpha_1/x_3 \cos(\nu \ln x_3 + \phi_1) + \beta_1 x_3 \\ \alpha_2/x_3 \cos(\nu \ln x_3 + \phi_2) + \beta_2 x_3 \\ x_3 \end{pmatrix}$$

where α_1 and α_2 are functions of the initial conditions and the vector $(\beta_1, \beta_2, 1)$ is the eigenvector of the matrix $\boldsymbol{A}$ with eigenvalue a_{33}. For nonzero α_i, the critical point on the surface will be avoided by all of the streamlines described by these equations. The only streamline that will intersect the surface at the spiral node is given by $\alpha_1 = \alpha_2 = 0$, and the direction of this line is determined by the eigenvector of $\boldsymbol{A}$ with eigenvalue a_{33}.

The vorticity field on the surface is generally nonzero and perpendicular to the limiting streamline field.[11] However, where there is a critical point in the limiting streamline pattern, there also is a critical point in the vorticity field and vice versa. The expansion for the vorticity field at the spiral node-saddle is

$$\omega = \boldsymbol{B}x$$

or

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} -a_{21} & -a_{22} & -a_{23} \\ a_{11} & a_{12} & a_{13} \\ 0 & 0 & a_{21} - a_{12} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

and the matrix $\boldsymbol{B}$ has an eigenvalue $a_{21} - a_{12}$. The equations for the vorticity lines also can be obtained, and the only vorticity line that will intersect the surface at the spiral node is the vorticity line whose direction at the surface is given by the eigenvector of $\boldsymbol{B}$ with eigenvalue $a_{21} - a_{12}$. Generally, the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are different, their eigenvectors differ, and the streamline and vorticity line exiting the surface at the spiral node will not be aligned. However, since the velocity and vorticity fields are perpendicular on the surface, the limit of the normalized helicity along the streamline that intersects the surface at the spiral node-saddle does represent an extremum (and discontinuity) in the normalized helicity.

Curvature and Normalized Helicity

For global separations, the alignment of the velocity and vorticity fields or the existence of an extremum in the normalized helicity at the streamline that defines the edge of a separation sheet does not imply that the curvature of this streamline is equal to zero or that it is at a minimum. Furthermore, it does not imply that the normalized helicity will have a maximum along the entire length of this streamline. Finding the location of minimum curvature and/or maximum normalized helicity downstream of the critical point and defining the vortex centerline by the streamline that passes through this point may not yield the same results as defining the vortex centerline by one of the streamlines exiting a spiral node-saddle in the flow (as for the horseshoe separation) or a spiral node on the surface (as for the Werle-type separation). However, for crossflow separation, there are no critical points that can be used to define the vortex centerline. Instead, definitions, such as a minimum in the streamline curvature or a maximum in the normalized helicity, must be used. Furthermore, these definitions also can be applied downstream of critical points for vortices resulting from global separations when the location and characteristics of the critical points are not precisely known.

The curvature of any line is given by

$$\kappa = \left| \frac{\mathrm{d}\hat{t}}{\mathrm{d}s} \right|$$

where $\hat{t}$ is the tangent vector to the curve and s is the arc length along the curve. For streamlines, the tangent vector is $\hat{u} = \boldsymbol{u}/|\boldsymbol{u}|$, and $\mathrm{d}/\mathrm{d}s$ can be written as $\hat{u} \cdot \nabla$. Manipulation of the curvature definition and use of the Navier-Stokes equations gives

$$\kappa = \frac{|\hat{u} \times \boldsymbol{u} \cdot \nabla \boldsymbol{u}|}{\boldsymbol{u} \cdot \boldsymbol{u}} = \frac{|-\nabla_n p + \rho \tau_n|}{\rho \boldsymbol{u} \cdot \boldsymbol{u}} = \frac{|f_n|}{\rho \boldsymbol{u} \cdot \boldsymbol{u}}$$

where $\nabla_n p$ and τ_n are the components normal to the streamline of the pressure gradient and the shear stress force per unit mass. The curvature of the streamline is a function of the kinetic energy of the fluid and f_n, the component of the total force normal to the path of the fluid particles. An extremum in the curvature exists only if f_n is zero (hence, the streamline has zero curvature) or the derivative normal to the streamline of $|f_n|$ is balanced by the derivative of the kinetic energy. Note that, because of the denominator, this equation cannot be used at critical points where the velocity magnitude is equal to zero.

In the very large or infinite Reynolds number case, the viscous forces are negligible, and the streamline curvature is a

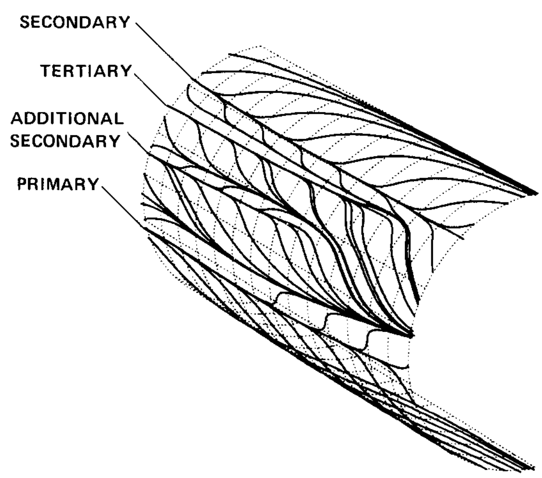

Fig. 3 Limiting streamline pattern six to eight body diameters aft of the nose.

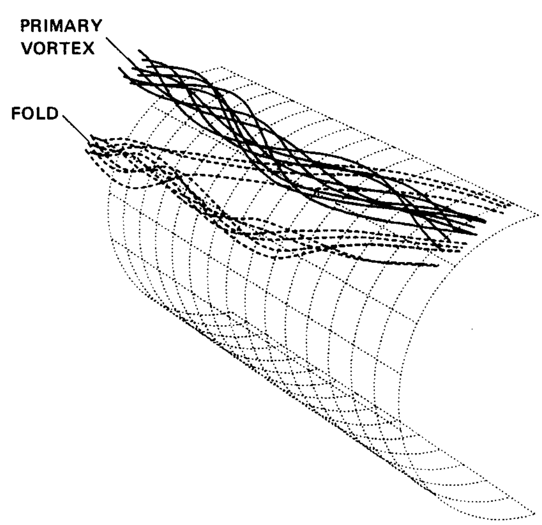

Fig. 4 Streamlines spiraling about a vortex centerline.

function of the pressure gradient. In this case, there is a pressure extremum at the line of minimum curvature if and only if the curvature of the streamline is equal to zero.

The normalized helicity, on the other hand, is simply the cosine of the angle between the velocity and vorticity fields. In Fig. 2, the vorticity direction is plotted along a streamline initially outside the vortex. As the streamline continues downstream, it is caught up in the rotating flow associated with the vortex. While the streamline is outside the vortex, the vorticity lines are approximately orthogonal to the streamline. On the surface, the limiting streamlines and the vorticity lines are indeed orthogonal.[11] As the streamline is caught up in the vortical flow, the angle between the vorticity field and the streamline becomes smaller until the two fields are almost parallel. Defining the vortex centerline by an extremum in the normalized helicity uses this characteristic of vortical flows. The normalized helicity is given by

$$H = \frac{\boldsymbol{u} \cdot \boldsymbol{\omega}}{|\boldsymbol{u}|\,|\boldsymbol{\omega}|}$$

and, trivially, it is equal to one if and only if the velocity and vorticity fields are aligned. It easily can be shown that

$$1 - |H| = \frac{|\hat{u} \times (\boldsymbol{u} \times \boldsymbol{\omega})|}{|\boldsymbol{u}|\,|\boldsymbol{\omega}|}$$

$$= \frac{|\hat{u} \times \boldsymbol{u} \cdot \nabla \boldsymbol{u} - \tfrac{1}{2}\hat{u} \times \nabla \boldsymbol{u} \cdot \boldsymbol{u}|}{|\boldsymbol{u}|\,|\boldsymbol{\omega}|}$$

$$= \frac{|\boldsymbol{u}|}{|\boldsymbol{\omega}|} \left| \frac{f_n}{\rho \boldsymbol{u} \cdot \boldsymbol{u}} - \frac{\nabla_n |\boldsymbol{u}|}{|\boldsymbol{u}|} \right|$$

where ∇_n represents only the gradient terms normal to the streamline. The normalized helicity is a function of the velocity magnitude, the vorticity magnitude, and the forces normal to the streamline. Expressing this equation in terms of curvature gives

$$1 - |H| = \frac{|\boldsymbol{u}|}{|\boldsymbol{\omega}|} \sqrt{\kappa^2 - \frac{\nabla_n |\boldsymbol{u}|}{\boldsymbol{u}} \cdot \left(2\frac{f_n}{\rho \boldsymbol{u} \cdot \boldsymbol{u}} - \frac{\nabla_n |\boldsymbol{u}|}{|\boldsymbol{u}|} \right)}$$

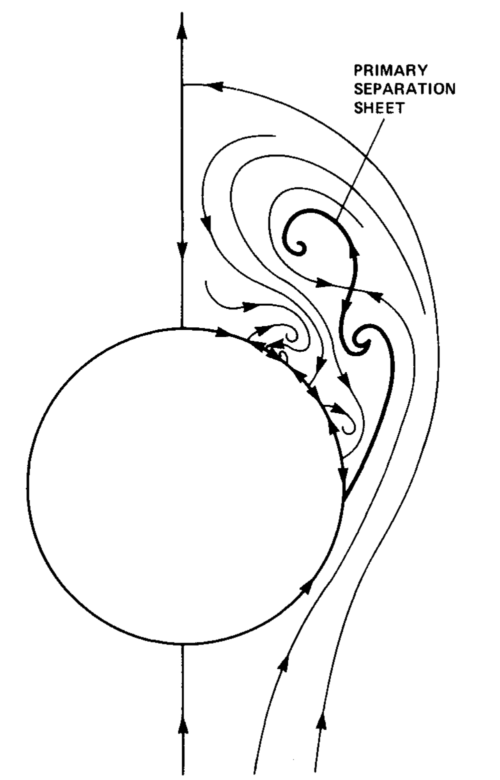

Fig. 5 Topological schematic in crossflow plane.

It is obvious from this expression that an extremum in curvature does not imply an extremum in the normalized helicity or vice versa.

This equation for the normalized helicity in terms of the curvature, forces, and velocity magnitude is fairly complicated, and its implications are not obvious. However, there are often extrema in the velocity magnitude, the vorticity magnitude, the density, and the pressure linked with vortices in the flow, and the assumption of these extrema can simplify the equations for the curvature, the normalized helicity, and their extrema.

Critical Points

For a critical point in a symmetry plane or on the surface, it has been shown that an extremum in the normalized helicity exists at the critical point. The velocity magnitude is zero and therefore at an extremum. Since the velocity is equal to zero, the curvature of the streamline is a function of higher order derivatives and is generally not equal to zero.

Extremum in the Velocity Magnitude

If an extremum in the velocity magnitude occurs at a point, an extremum in the curvature will occur if and only if there is an extremum in the magnitude of the normal force per unit mass $|f_n|/\rho$. The equation for the normalized helicity reduces to

$$1 - |H| = \frac{|\boldsymbol{u}|\kappa}{|\boldsymbol{\omega}|}$$

and an extremum in the normalized helicity will occur when

$$\hat{u} \times \nabla |H| = 0$$

or

$$\frac{1}{|\boldsymbol{\omega}|} \left[|\boldsymbol{u}| \nabla_n \kappa - \frac{f_n \cdot \nabla_n}{\rho |\boldsymbol{u}|^2} \nabla_n |\boldsymbol{u}| - |\boldsymbol{u}| \kappa \frac{\nabla_n |\boldsymbol{\omega}|}{|\boldsymbol{\omega}|} \right] = 0$$

Note that this does not guarantee coincidental extremum in curvature and normalized helicity; the last two terms are not necessarily zero when the curvature is at a minimum.

Extremum in the Vorticity Magnitude

An extremum in the vorticity magnitude alone does not provide any additional information about the curvature of the line. The equation for an extremum in the normalized helicity is reduced to

$$\frac{1}{|\boldsymbol{\omega}|} \left[|\boldsymbol{u}| \nabla_n \kappa - \frac{f_n \cdot \nabla_n}{\rho |\boldsymbol{u}|^2} \nabla_n |\boldsymbol{u}| \right] = 0$$

Again, the extrema in the curvature and normalized helicity need not coincide.

Zero Curvature

When the curvature is equal to zero, the normal forces per unit mass are equal to zero. The equation for the normalized helicity reduces to

$$1 - |H| = \frac{|\nabla_n |\boldsymbol{u}||}{|\boldsymbol{\omega}|}$$

This implies that the velocity and vorticity fields need not be aligned when the curvature of the streamline is zero, and, furthermore, an extremum in the normalized helicity need not lie on a streamline of zero curvature.

Extremum in the Velocity Magnitude and Zero Curvature

When an extremum in the velocity magnitude coincides with a streamline of zero curvature, the absolute value of the normalized helicity is equal to one. Hence, a combination of a minimum in the curvature and an extremum in the velocity magnitude correlates with a maximum in the normalized helicity. Under these conditions, defining a vortex centerline using either method will give identical results.

Streamlines and Vortex Centerlines

If a streamline does coincide with extrema in a function $g(u, \omega)$, such as curvature or normalized helicity, certain criteria must be met. The first is that the derivatives normal to the streamline are zero; it is this criterion that was discussed previously. The second is that the derivatives normal to the streamline remain zero along the streamline, i.e.,

$$\hat{u} \cdot \nabla[\hat{u} \times \nabla g(u, \omega)] = 0$$

or

$$(\hat{u} \cdot \nabla \hat{u}) \times \nabla g + \hat{u} \times (\hat{u} \cdot \nabla) \nabla g = 0$$

Note that the first term contains curvature terms and the second term contains higher order derivatives of the function g. This equation can be manipulated to give the following criteria for coincident streamlines and extrema:

$$\kappa = \frac{|\hat{u} \times (\hat{u} \cdot \nabla) \nabla g|}{|\hat{u} \cdot \nabla g|}$$

If a single streamline is to coincide with the vortex centerline defined either by a minimum in the curvature or by a maximum in the normalized helicity, the curvature of that streamline is dictated by higher order derivatives of these functions.

Limitations on the forces, velocity magnitude, and vorticity magnitude can be obtained by replacing g with its functional form; however, these equations rapidly become complex. There are, however, some simplifying assumptions that reduce the complexity of these equations.

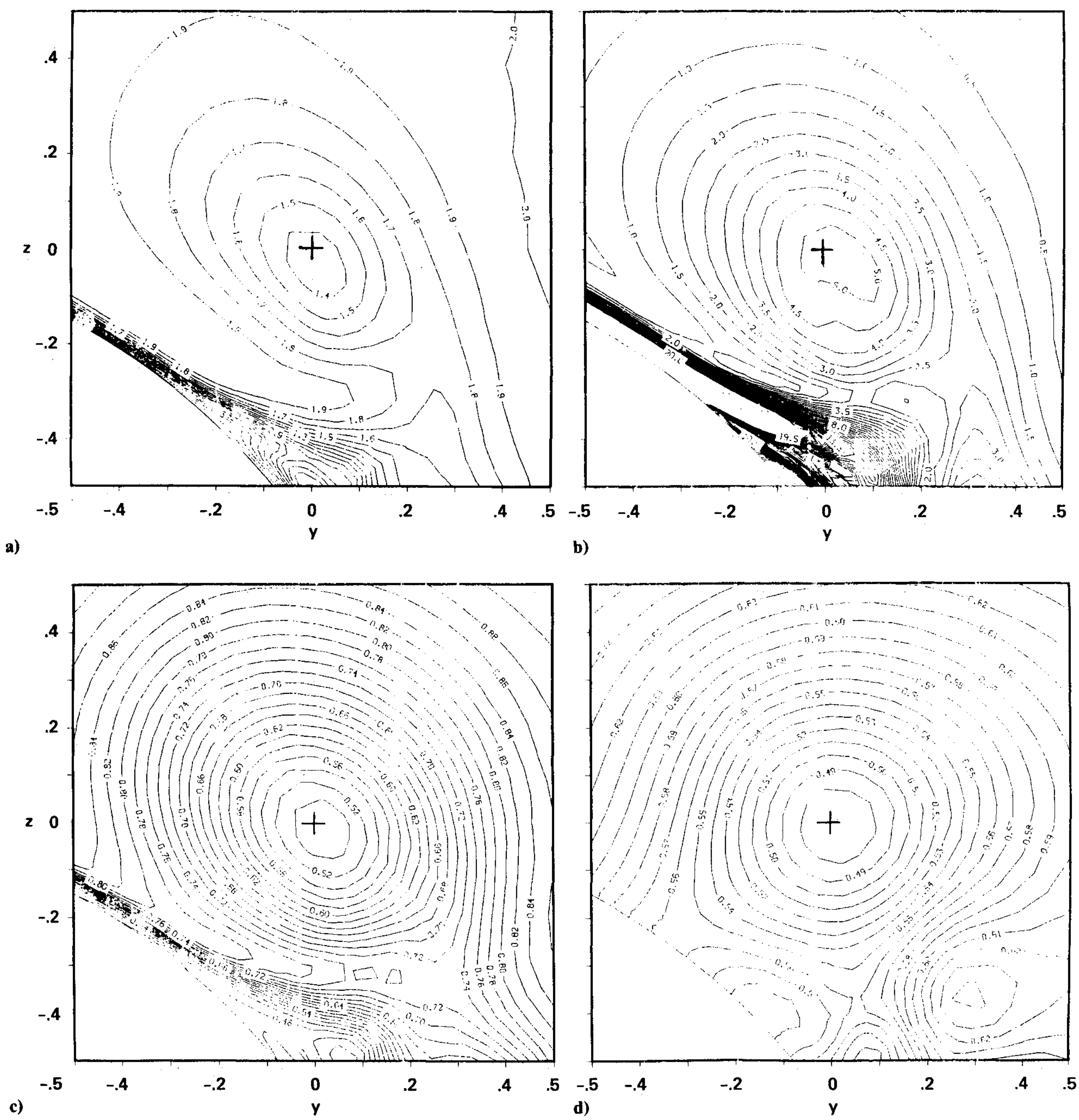

Fig. 6 Examples of extrema at the vortex centerline. Origin is location of the streamline with minimum curvature; a) velocity, b) vorticity, c) density, d) pressure.

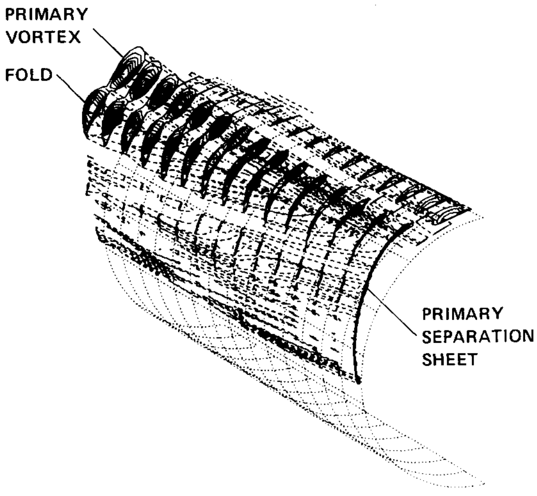

Fig. 7 Normalized helicity.

Normalized Helicity Equal to One

If the normalized helicity at the vortex centerline is equal to one along a length of the centerline, the velocity and vorticity fields must be aligned along the centerline. If a streamline coincides with this centerline, then

$$u \times \omega = 0$$

and

$$\hat{u} \cdot \nabla(u \times \omega) = 0$$

Manipulation of these equations gives

$$\hat{u} \times (u \cdot \nabla\omega - \omega \cdot \nabla u) = 0$$

and a streamline is coincident with the alignment of the velocity and vorticity fields only if the rate of change normal to the streamline of the vorticity following a fluid mass is due only to the straining of the vortex lines. If the production and dissipation of vorticity normal to the streamline from viscous and pressure sources are not equal, the vorticity and velocity fields will not remain aligned.

In the infinite Reynolds number limit for barotropic flow, the only production of vorticity off the surface is due to the straining of the vortex lines, and if the normalized helicity is equal to one at some point on a streamline, it is equal to one along the entire line. When this streamline begins at a spiral node-saddle in a symmetry plane, the entire vortex centerline defined by an extremum in the normalized helicity coincides with the edge of the separation sheet and, therefore, a streamline that exits the symmetry plane at the spiral node-saddle.

Curvature Equal to Zero

In many cases it appears that a single streamline is at the center of the vortex and that the curvature of this streamline is nearly zero. If this is the case, the equations for curvature can be manipulated to give $\hat{u} \cdot \nabla(f_n/\rho) = 0$, or the force per unit mass normal to the streamline remains zero along the length of the vortex centerline.

Summary of Analytical Results

A vortex centerline defined by a maximum in the normalized helicity will coincide with a vortex centerline defined by a minimum in the curvature and with a streamline only under very restrictive conditions. This has not only been shown here analytically but also in the computations of Levy et al.[7] In their example of a computed vortical flowfield for a hemisphere cylinder, the centerline was defined by a streamline that had maximum normalized helicity at some point downstream of the critical point. This streamline was then traced back to the vicinity of a critical point in the flow; however, as this streamline was traced back to the critical point, it deviated from an almost straight line and began to spiral about another streamline. In this region a maximum in the normalized helicity or a minimum in the curvature would have defined a different streamline as the centerline of the vortex.

Computational Results

In the preceding section, the differences between defining a vortex centerline by a minimum in the curvature and a maximum in the normalized helicity were discussed. Furthermore, it was shown that one single streamline need not coincide with either definition. However, under certain conditions, the two extrema do define the same line and this line is a streamline. It is impossible to determine whether these conditions are met by using analysis alone, and the Navier-Stokes equations must be solved. Levy et al.[7] performed an extensive computational study of vortices and the normalized helicity definition of a vortex centerline. However, they assumed that a maximum in the normalized helicity was equivalent to a minimum in the curvature and did not show this computationally. In this computational study, vortex centerlines are defined by calculating the curvature of the spiraling streamlines in the vortex and choosing the streamline of minimum curvature. Furthermore, planes normal to the vortex centerline, not computational planes, are used to locate extrema in the flowfield parameters.

A parabolized Navier-Stokes code based on the Rai and Chaussee refinement[12] of the Schiff-Steger algorithm[13] was used to solve the Navier-Stokes equations and to compute vortical flows formed by crossflow separation from a simple configuration, namely that of a tangent ogive cylinder. The gridding scheme of the code was modified so that features of both the crossflow separation and the vortical flow could be better resolved; details of the modification are discussed in Yates and Chapman.[10] The conditions chosen for the computations were laminar flow, a freestream velocity of Mach 2, a 10-deg angle of attack, and a Reynolds number based on diameter of 100,000.

Streamlines and Extrema

In Fig. 3, the limiting streamline pattern is shown for a portion of the tangent ogive cylinder that is six to eight body diameters aft of the nose. This limiting streamline pattern has several separation and reattachment lines and is consistent with experimentally observed patterns. The crossflow patterns, not shown here, are also consistent with experimentally observed patterns. At this distance from the nose, the flow is very complicated. The limiting streamline pattern and the crossflow velocity patterns indicate that there are primary,

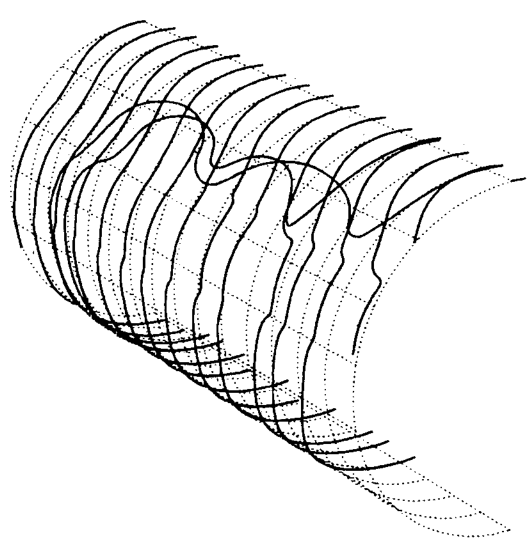

Fig. 8 Vorticity lines; simple closed paths.

secondary, and tertiary crossflow separations. The vortices in the flow associated with these separations are readily identified by organized regions of flow in which the streamlines spiral about a central core, as illustrated for the primary separation in Fig. 4.

For these flow conditions and at this distance from the nose, there are thin sheets associated with the primary separation where the vorticity magnitude reaches a maximum. Although these sheets do not satisfy the definition of a separation sheet (a sheet defined by the streamlines exiting the surface at a saddle point and extending into the flow), this far aft of the onset of the crossflow separation, these thin sheets do appear to have many of the characteristics of a separation sheet and will be called separation sheets for simplicity. These sheets develop folds that subsequently roll up. These folds and their subsequent roll up have been observed experimentally by Intemann.[14] The computations further indicate that these folds and their roll up induce additional secondary vortices near the surface that have the same rotational sense as the initial secondary vortices. A topological schematic of the initial primary, secondary, and tertiary separations, the folds on the separation sheet, and the additional secondaries are shown in Fig. 5.

For crossflow separation, there are no critical points on the surface of the ogive cylinder. Hence, there are no spiral nodes that are identified with the intersection of the vortex and the surface, and the vortex centerline cannot be defined by a streamline that intersects the surface at a spiral node (as is the case for Werle-type separation). Furthermore, the vortices do not pass through a plane of symmetry, and the vortex centerline cannot be defined by a spiral node in a symmetry plane (as is the case for a horseshoe vortex). Here, the vortex centerline has been defined by a streamline within the region of spiraling streamlines that has minimum curvature.

One streamline may not be sufficient to define the entire length of the vortex centerline; instead, small segments from several streamlines may be necessary to define the entire centerline. For example, the centerline of a vortex that is induced to change direction by the shock wave produced by a flare is defined by segments of two streamlines. Computations indicated that the streamline with minimum curvature before the vortex passed through the shock wave followed a corkscrew path about another streamline after the shock. Both streamlines remained in the finite region defined as the vortex core. Although this is an extreme case, any bend in the vortex centerline may cause some discrepancy in the curvature and normalized helicity definitions of the centerline, and these two definitions may not correspond with one single streamline. However, after the bend, the definitions may again coincide with each other and with one single streamline. When the vortex centerline is defined using the minimum curvature definition, the computations indicate that several extrema in the flowfield parameters occur at the centerline (within a grid point). These extrema should be properly demonstrated in planes perpendicular to the centerline, not in computational planes. For the cases illustrated in Fig. 6, the pressure, density, velocity magnitude, and vorticity magnitude are plotted in planes perpendicular to the streamline with minimum curvature. It should be noted that this far aft of the nose, this streamline was nearly straight. For streamlines with zero curvature, the analysis indicated that the normalized helicity would be equal to one if and only if the velocity magnitude was at an extremum. The computations do indicate that there is an extremum in the velocity magnitude at or near the centerline and that the normalized helicity is equal to one. Therefore, the centerlines defined by minimum curvature and maximum normalized helicity should coincide. If the centerline was bending and/or the velocity magnitude did not have an extremum on the line of minimum curvature, the centerlines defined by minimum curvature and maximum normalized helicity would be offset from each other. Other computations that have indicated these extrema have used computational planes, not normal planes (e.g., Ying et al.[9] and Levy et al.[7]); this could result in some inaccuracy in locating the actual positions of the extrema.

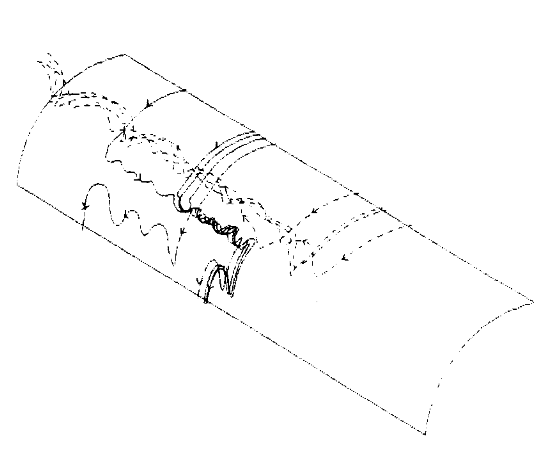

Fig. 9 Vorticity lines; complicated closed paths.

Vorticity and Normalized Helicity

In Fig. 7 the angles between the vorticity and velocity fields are plotted. In the regions defined as the centerlines of the vortices, the angles between the two fields are nearly 0 or 180 deg. These regions are very localized, and the locations of the centerlines are well defined, including the secondary rolling up of the separation sheet. The vorticity and velocity fields are also nearly parallel in very thin sheets that are associated with the separation. These sheets correspond to a maximum in the vorticity magnitude. It should be noted that for crossflow separation, the velocity and vorticity fields remain nearly aligned along a streamline in the vortex core region.

The computations showed that this near alignment of the velocity and vorticity fields can be used to locate the vortex centerline, and it represents the basis of Levy's[7] graphical representation. The alignment of these two fields only in localized regions of the flow is reflected in the behavior of the vorticity lines. Outside the immediate region of the vortex, the vorticity is predominantly perpendicular to the velocity field, and many of the vorticity lines form simple closed paths about the body (Fig. 8). However, some of the vorticity lines follow complicated closed paths. For instance, in Fig. 8 there are vorticity lines that begin at the leeward side and are predominantly perpendicular to the velocity field. As they near the core of the primary vortex, they bend abruptly and become parallel to the core. The number of turns per length along the vortex core and the direction of the turning is not, in general, equal to that of the streamlines. Furthermore, diffusive effects cause the vorticity lines to move slowly away from the core whereas the streamlines spiral into the core. After spiraling about the vortex core for a finite distance downstream, the vorticity lines bend suddenly and again are almost perpendicular to the velocity field. They then continue around the body to the windward side. Since the flows studied here assume bilateral symmetry, they connect with their mirror images and form closed loops.

In the computed flowfields, more complicated loops can and do occur. For instance, one of the vorticity lines in Fig. 9 first follows the core of the secondary vortex; it then turns and follows the core of the vortex associated with the fold on the primary separation sheet. However, no matter how many twists and turns the computed vorticity lines take, they eventually form closed loops. This implies that the vorticity line pattern for crossflow separation can be obtained by simply stretching and twisting the pattern for attached flow; no cuts or breaks of the lines would be necessary. For local separation, this is topologically consistent.

Conclusions

The experimentally observed features of crossflow separation and the resulting vortical flows associated with it, including the development of folds on the primary separation sheet and the subsequent roll-up of these folds, can be successfully simulated by parabolized Navier-Stokes flowfield computations. It is therefore possible to use these detailed computed flowfields to gain insight into the structure of vortical flows.

For steady global separation, a vortex centerline can be precisely defined by the edge of a separation sheet. For crossflow separation, there is no equivalent definition. However, the visual identification of a three-dimensional vortex as a region of spiraling streamlines suggests an alternative definition for the vortex centerline: a streamline within a region of spiraling streamlines that has the minimum curvature. Theoretical analysis indicates that this definition does not necessarily coincide with other definitions, such as extrema in the normalized helicity, velocity magnitude, vorticity magnitude, or other flow parameters. However, in the infinite Reynolds number case, it has been shown that the vortex centerline defined by an extremum in the normalized helicity does agree with the edge of a separation sheet or, equivalently, a streamline exiting a spiral node-saddle in the flow. In this case, an extremum in the pressure occurs at the vortex centerline only if the centerline is straight.

From computed flowfields, it was found that when the vortex centerline was defined a few body diameters aft of the onset of separation by a streamline with minimum curvature, this streamline had nearly zero curvature, and extrema in the density, pressure, velocity magnitude, vorticity magnitude, and normalized helicity occurred at the vortex centerline (within a grid point). Extrema in the normalized helicity also occurred on surfaces that could be identified with the separation sheets. This agreement of the position of the extrema and the lines of minimum curvature may be a consequence of the large sectional Reynolds number; we may be approaching the infinite Reynolds number limit. In addition, for crossflow separation, it has been shown that the calculated vortex lines form closed paths about the projectile.

Acknowledgments

Most of this work was completed while G. T. Chapman was a senior staff scientist at NASA Ames Research Center and L. A. Yates was a research associate with the National Research Council. Further support for L. A. Yates was provided by NASA Grant NCC 2-583.

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